Bayesian Inference for Symmetry Energy

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under Controlled Conditions

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Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under $n \leftrightarrow p$ interchange

An isoscalar quantity F does not change under $n \leftrightarrow p$ interchange. E.g. nuclear energy. Expansion in asymmetry $\eta = (N - Z)/A$, for smooth F, yields even terms only:

$$F(\eta) = F_0 + F_2\,\eta^2 + F_4\,\eta^4 + \dots$$

An isovector quantity G changes sign. Example: $\rho_{np}(r) = \rho_n(r) - \rho_p(r)$. Expansion with odd terms only:

$$G(\eta) = G_1\,\eta + G_3\,\eta^3 + \dots$$

Note: $G/\eta = G_1 + G_3\,\eta^2 + \dots$.

In nuclear practice, analyticity requires shell-effect averaging!

Charge invariance: invariance of nuclear interactions under rotations in n-p space

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Charge invariance:
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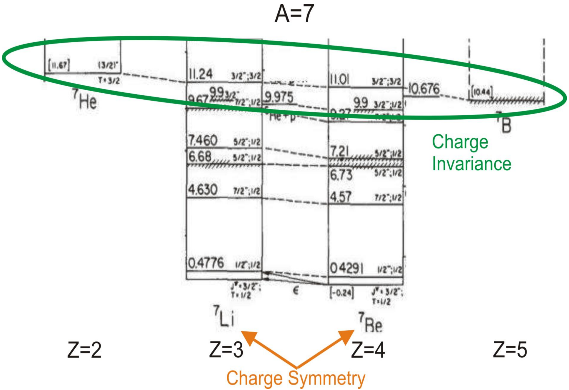
Isospin doublets

$$p : (\tau, \tau_z) \quad = (\tfrac{1}{2}, \tfrac{1}{2})$$
$$n : (\tau, \tau_z) \quad = (\tfrac{1}{2}, -\tfrac{1}{2})$$

Net isospin

$$\vec{T} = \sum_{i=1}^{A} \vec{\tau}_i$$

Isobars: Nuclei with the same *A*



$T = \frac{3}{2}, \ldots \qquad T = \frac{1}{2}, \frac{3}{2}, \ldots \qquad T = \frac{3}{2}, \ldots$

Nuclear states: $(T, T_z), \quad T \geq |T_z| = \frac{1}{2}|N - Z|$

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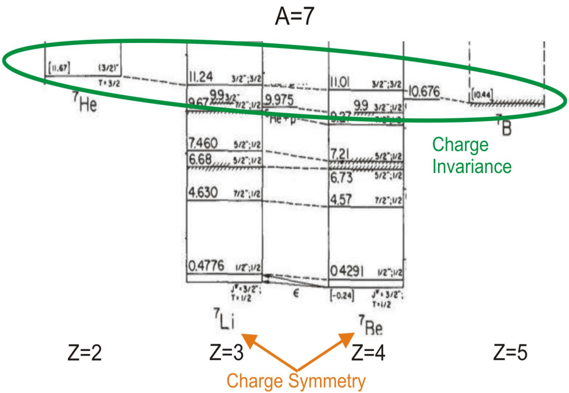

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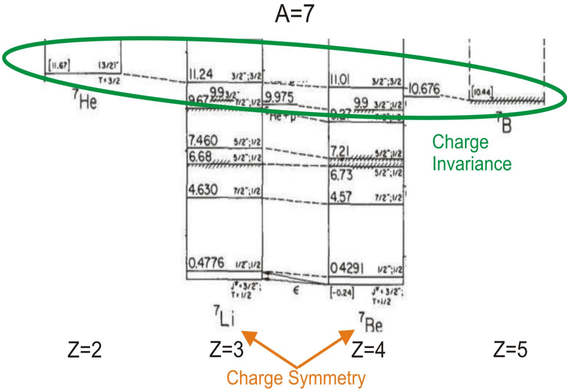

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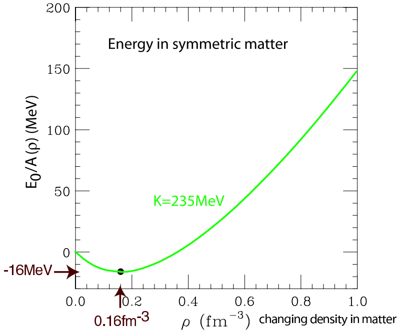
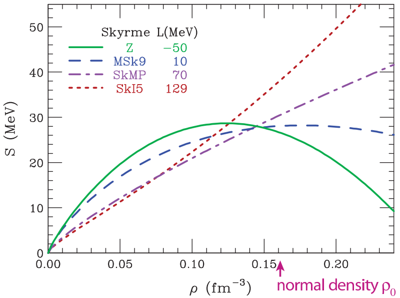
Energy in Uniform Matter

$$\frac{E}{A}(\rho_n, \rho_p) = \frac{E_0}{A}(\rho) + S(\rho)\left(\frac{\rho_n - \rho_p}{\rho}\right)^2 + \mathcal{O}(\dots^4)$$

symmetric matter (a)symmetry energy $\rho = \rho_n + \rho_p$





$$\frac{E_0}{A}(\rho) = -a_V + \frac{K}{18}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \dots \qquad S(\rho) = a_a^V + \frac{L}{3}\frac{\rho - \rho_0}{\rho_0} + \dots$$

Known: $a_a \approx 16\,\text{MeV}$ $K \sim 235\,\text{MeV}$ Unknown: a_a^V ? L ?

Isoscalar and Isovector Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar $\Rightarrow$ weakly depends on $(N - Z)$ for given A. [Coulomb suppressed. . .]

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A\,\rho_{np}(r)/(N - Z)$ isoscalar!

$A/(N - Z)$ normalizing factor global. . . Similar local normalizing factor, in terms of intense quantities, $2a_a^V/\mu_a$, where $a_a^V \equiv S(\rho_0)$

Isoscalar formfactor for isovector density:

$$\rho_a(r) = \frac{2a_a^V}{\mu_a}\left[\rho_n(r) - \rho_p(r)\right]$$

Normal matter: $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on η!

In any nucleus:

$$\boxed{\rho_{n,p}(r) = \frac{1}{2}\left[\rho(r) \pm \frac{\mu_a}{2a_a^V}\rho_a(r)\right]}$$

where $\rho(r)$ & $\rho_a(r)$ have universal features! (subject to shell effects)

No shell-effects, ρ's as dynamic vbles: Hohenberg-Kohn functional

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Net density ρ usually parameterized w/Fermi function

$$\rho(r) = \frac{\rho_0}{1+\exp\left(\frac{r-R}{d}\right)} \qquad \text{with} \quad R = r_0\, A^{1/3}$$

Isovector density ρ_a?? Related to $S(\rho)$!

In uniform matter

$$\mu_a = \frac{\partial E}{\partial(N-Z)} = \frac{\partial[S(\rho)\,\rho_{np}^2/\rho]}{\partial\rho_{np}} = \frac{2\,S(\rho)}{\rho}\,\rho_{np}$$

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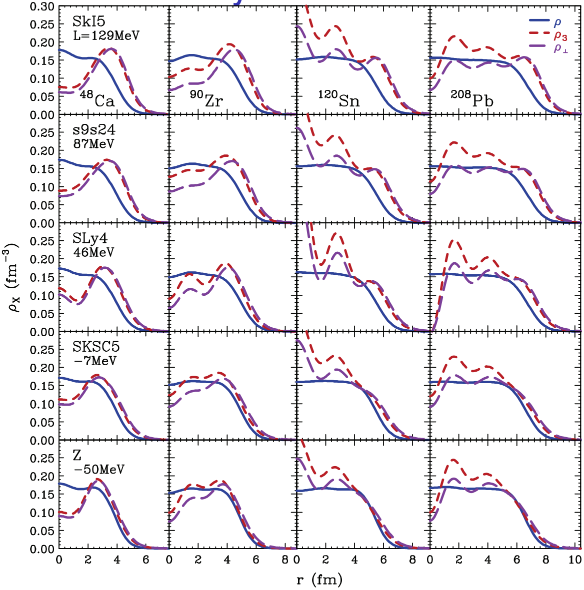


$\rho = \rho_n + \rho_p$

$\rho_3 \propto (\rho_n - \rho_p)$

$\rho_\perp \equiv \rho_a$: Coulomb-corrected ρ_3 density f/pure isospin state

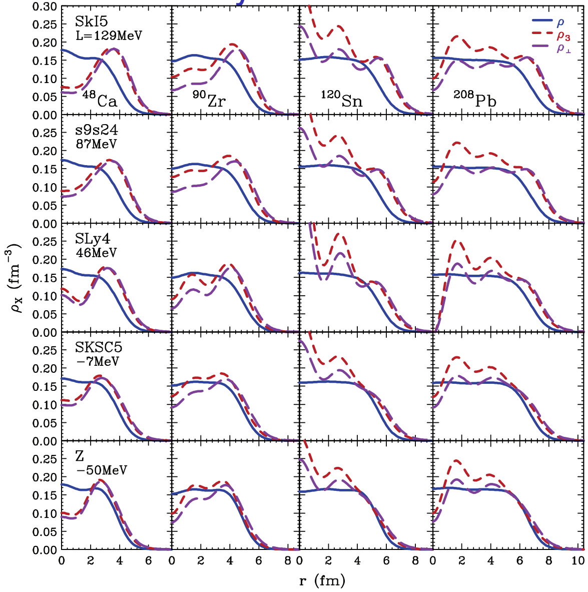
$\leftrightarrow$ same interaction

$\updownarrow$ same nucleus

Surface ~same f/every nucleus

The higher L, the farther isovector & isoscalar surfaces split apart

Skyrme-Hartree-Fock Densities



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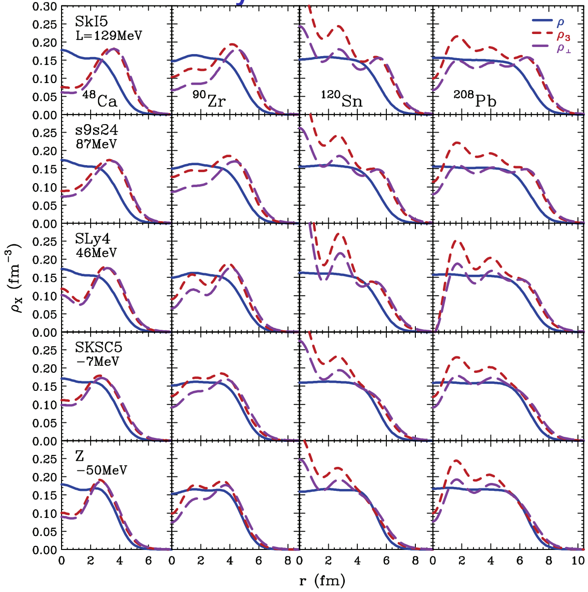

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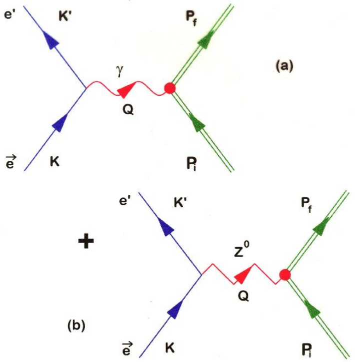
Strategies for Independent Densities

Jefferson Lab

Direct: $\sim p$

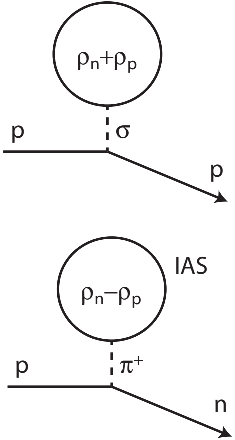
Interference: $\sim n$



PD

elastic: $\sim p + n$

charge exchange: $\sim n - p$



Why Isovector Aura Rather than Neutron Skin

Isovector aura: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!

Not suppressed by low $(N-Z)/A$!

Nucleon (Lane) optical potential in isospin space:

$$U = U_0 + \frac{4\tau T}{A} U_1$$

isoscalar potential $U_0 \propto \rho$, isovector potential $U_1 \propto (\rho_n - \rho_p)$

In elastic scattering $U = U_0 \pm \frac{N-Z}{A} U_1$

In quasielastic charge-exchange (p,n) to IAS: $U = \frac{4\tau_- T_+}{A} U_1$

Elastic scattering dominated by U_0

Quasielastic governed by U_1

Geometry usually assumed the same for U_0 and U_1

e.g. Koning & Delaroche NPA713(03)231

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In elastic scattering $U = U_0 \pm \frac{N-Z}{A} U_1$

In quasielastic charge-exchange (p,n) to IAS: $U = \frac{4\tau_- T_+}{A} U_1$

Elastic scattering dominated by U_0

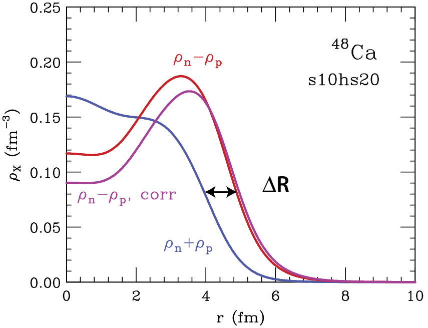
Quasielastic governed by U_1

Geometry usually assumed the same for U_0 and U_1

e.g. Koning & Delaroche NPA713(03)231

?Isovector aura ΔR from comparison of elastic and quasielastic (p,n)-to-IAS scattering?

Expectations on Isovector Aura?



Much Larger Than Neutron!

Surface radius $R \simeq \sqrt{\frac{5}{3}}\, \langle r^2 \rangle^{1/2}$

rms neutron skin

$$\langle r^2 \rangle^{1/2}_{\rho_n} - \langle r^2 \rangle^{1/2}_{\rho_p}$$

$$\simeq 2\,\frac{N-Z}{A}\left[\langle r^2 \rangle^{1/2}_{\rho_n - \rho_p} - \langle r^2 \rangle^{1/2}_{\rho_n + \rho_p}\right]$$

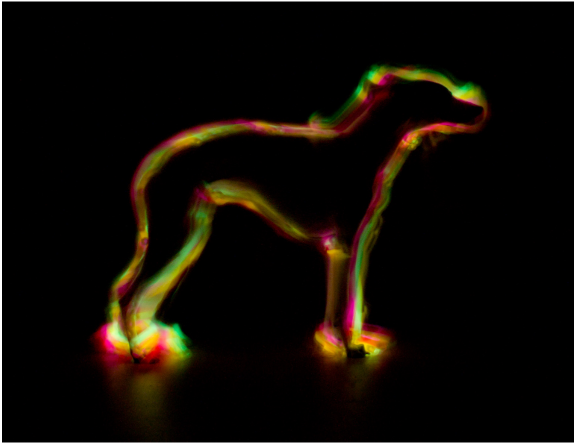
rms isovector aura

Estimated $\Delta R \sim 3\left(\langle r^2 \rangle^{1/2}_{\rho_n} - \langle r^2 \rangle^{1/2}_{\rho_p}\right)$ for ^{48}Ca/^{208}Pb!

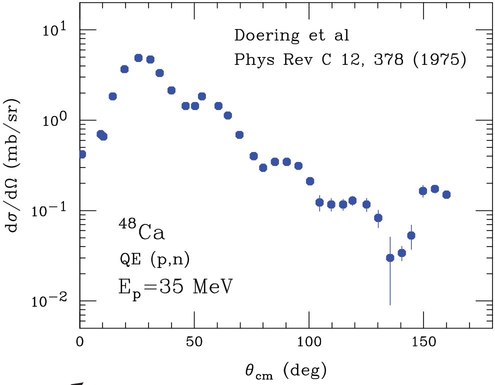
Even before consideration of Coulomb effects that further enhances difference!

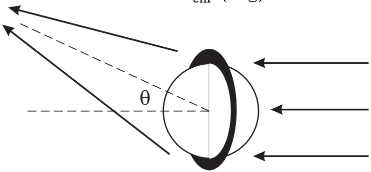
Aura

Historically Kirlian/Aura Photography



Direct Reaction Primer





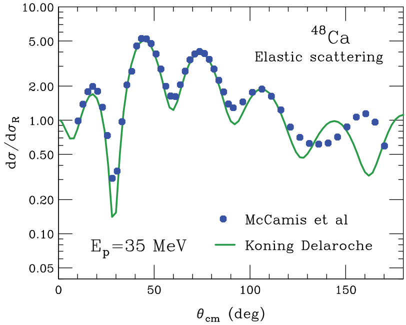
DWBA:

$$\frac{d\sigma}{d\Omega} \propto \left| \int dr\, \Psi_f^* \, U_1 \, \Psi_i \right|^2$$

- Oscillations: 2-side interference/source size
- Fall-off: softness of source
- Filling of minimae: imaginary/real contributions, spin-orbit

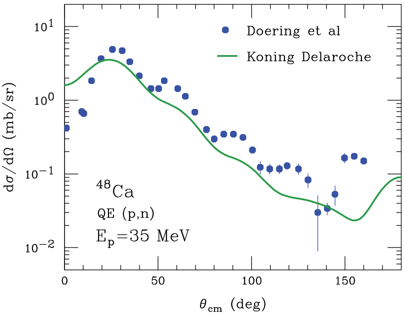
Potentials Fit to Elastic in Quasielastic

E.g. Koning-Delaroche NPA713(03)231 same radii for neutrons/protons, isoscalar/isovector, focus on p elastic



p Elastic Scattering

$$U_0 + \frac{N-Z}{A} U_1$$



QuasiElastic (p,n)

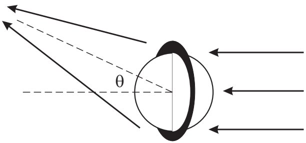
U_1 only?

Effect of Changing Isovector Radius

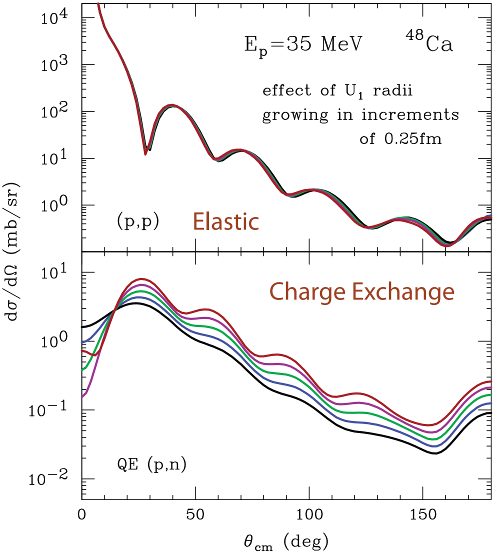
Koning-Delaroche
NPA713(03)231
same radii R for U_0 & U_1!

$$U_1(r) \propto \frac{U_{01}}{1 + \exp \frac{r-R}{a}}$$

$$R \rightarrow R + \Delta R_1$$



charge-exchange cs
oscillations grow

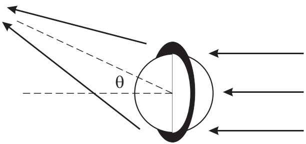


Effect of Changing Isoscalar Radius

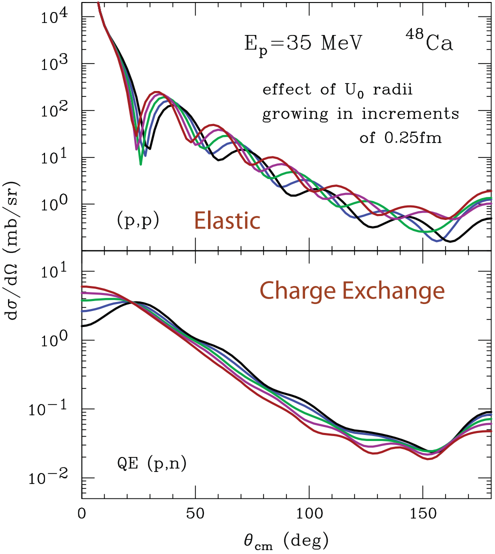
Koning-Delaroche
NPA713(03)231
same radii R for U_0 & U_1!

$$U_0(r) \propto \frac{U_{00}}{1 + \exp\frac{r-R}{a}}$$

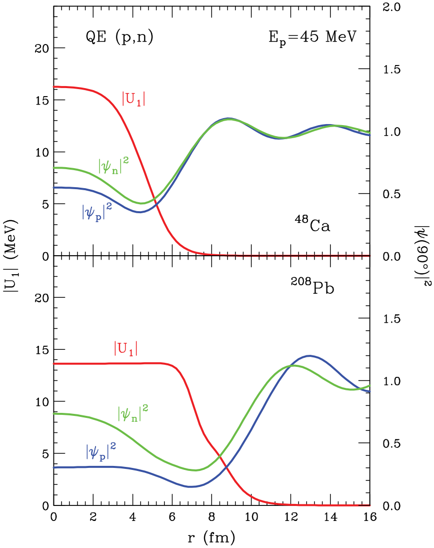
$R \rightarrow R + \Delta R_0$



charge-exchange cs
oscillations shrink



Impact of *U*-Radii on (p,n) Cross Section



DWBA

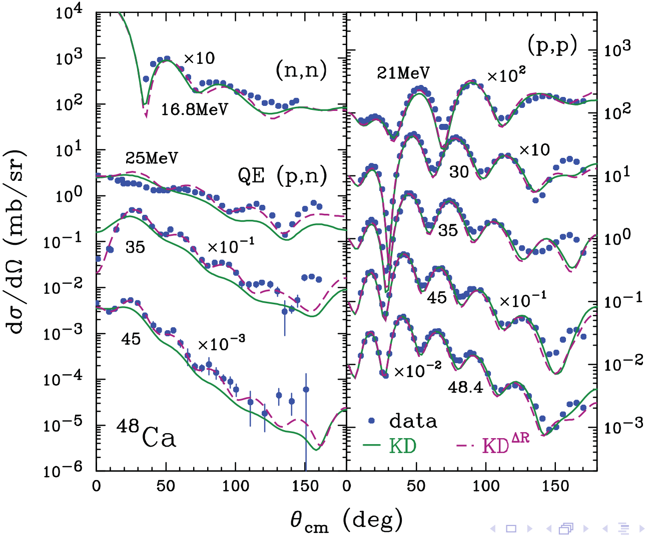
$$\frac{d\sigma}{d\Omega} \propto \left| \int dr\, \Psi_p^*(r)\, U_1(r)\, \Psi_n(i) \right|^2$$

Isoscalar radius responsible for holes in wavefunctions Ψ

Isovector radius responsible for region where (p,n) conversion can occur

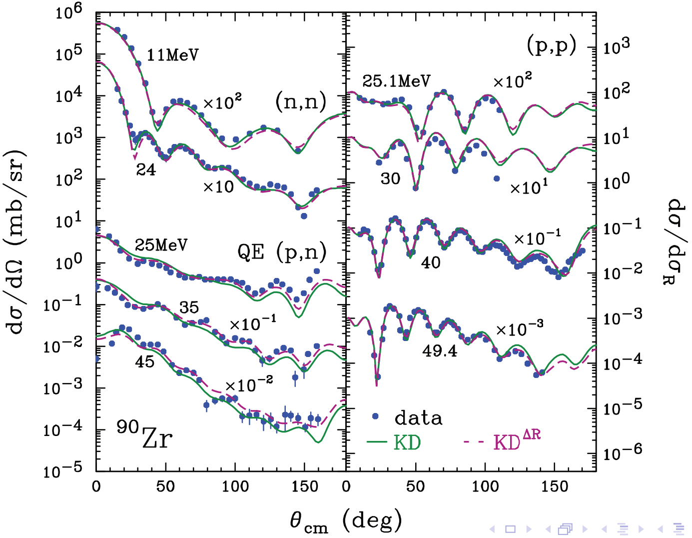
Modified Koning-Delaroche Fits: ^{48}Ca

In Koning-Delaroche: $R_{0,1} = R + \Delta R_{0,1}$ $a_{0,1} = a + \Delta a_{0,1}$



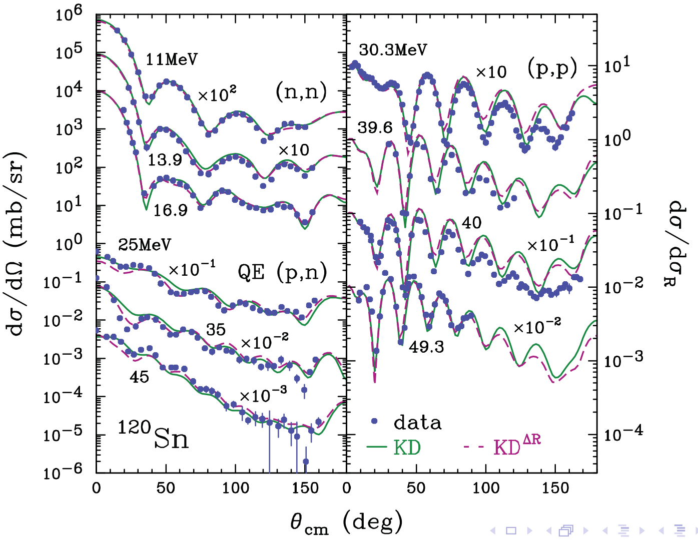
Modified Koning-Delaroche Fits: ^{90}Zr

In Koning-Delaroche: $R_{0,1} = R + \Delta R_{0,1}$ $\quad a_{0,1} = a + \Delta a_{0,1}$



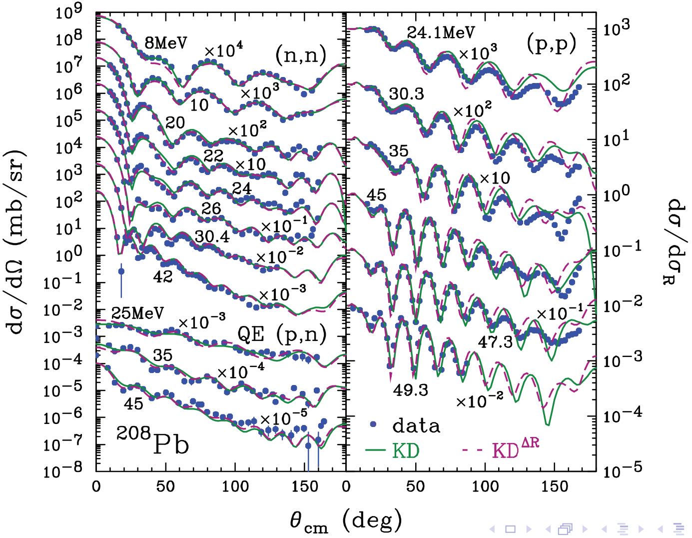
Modified Koning-Delaroche Fits: ^{120}Sn

In Koning-Delaroche: $R_{0,1} = R + \Delta R_{0,1}$ $a_{0,1} = a + \Delta a_{0,1}$

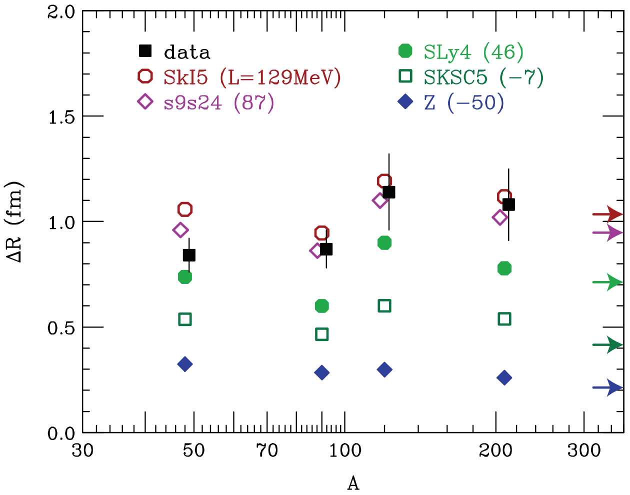


Modified Koning-Delaroche Fits: ^{208}Pb

In Koning-Delaroche: $R_{0,1} = R + \Delta R_{0,1}$ $a_{0,1} = a + \Delta a_{0,1}$



Thickness of Isovector Aura



Colored: Skyrme predictions. Arrows: half-infinite matter

Large ∼ 0.9 fm skins! ∼Independent of *A*...

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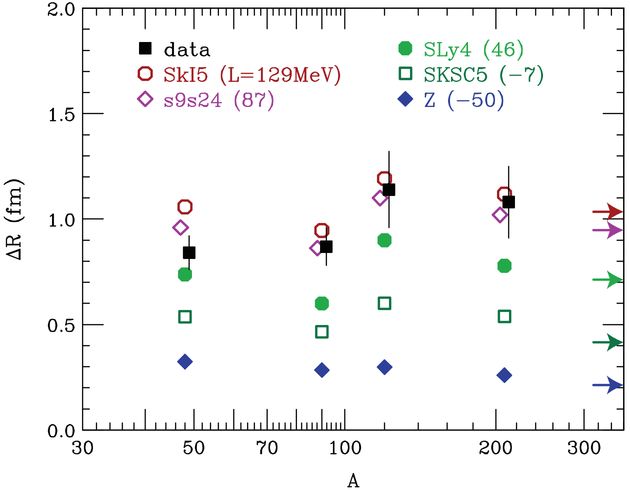

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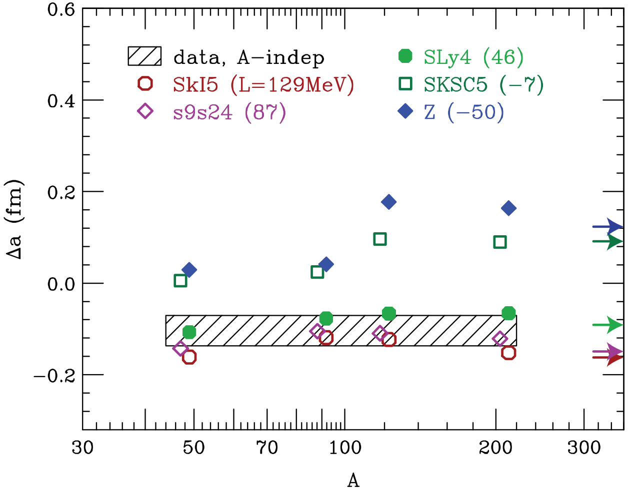
Large ~ 0.9 fm skins! $\sim$Independent of A...

Difference in Surface Diffuseness



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Sharper isovector surface than isoscalar!

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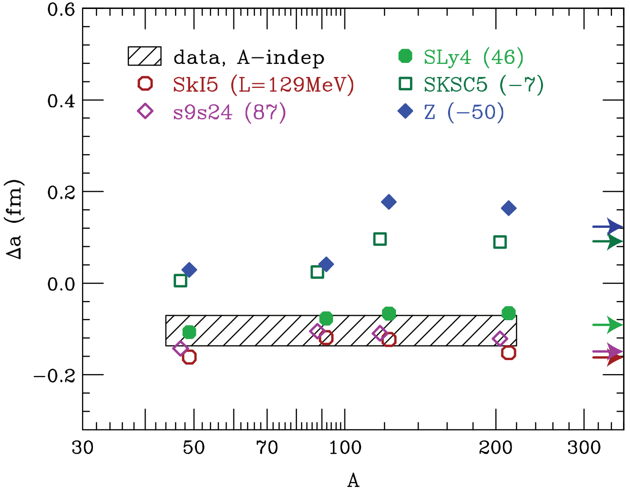

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Bayesian Inference

Probability density in parameter space $p(x)$ updated as experimental data on observables E, value $\overline{E}$ with error σ_E, get incorporated

Probability p is updated iteratively, starting with prior p_{prior}

$p(a|b)$ - conditional probability

$$p(x|\overline{E}) \propto p_{\text{prior}}(x) \int \mathrm{d}E \, \mathrm{e}^{-\frac{(E-\overline{E})^2}{2\sigma_E^2}} \, p(E|x)$$

For large number of incorporated data, p becomes independent of p_{prior}

In here, p_{prior} and $p(E|x)$ are constructed from all Skyrme ints in literature, and their linear interpolations. p_{prior} is made uniform in plane of symmetry-energy parameters (L, a_a^V)

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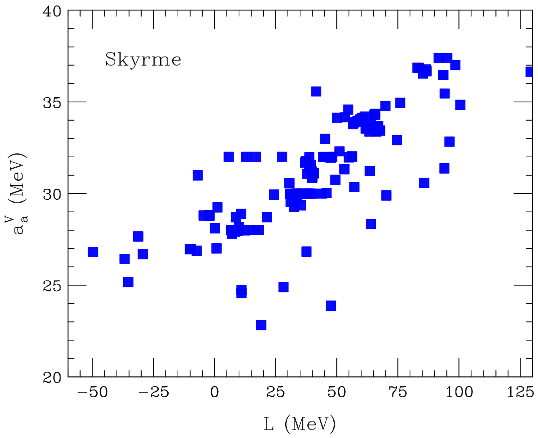
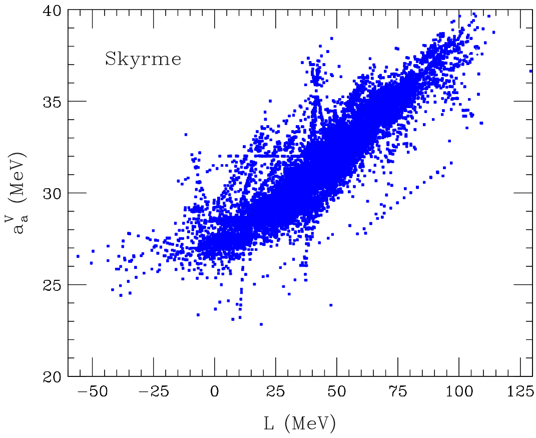
Raw Skyrme Parametrizations in (a_a^V, L) Plane

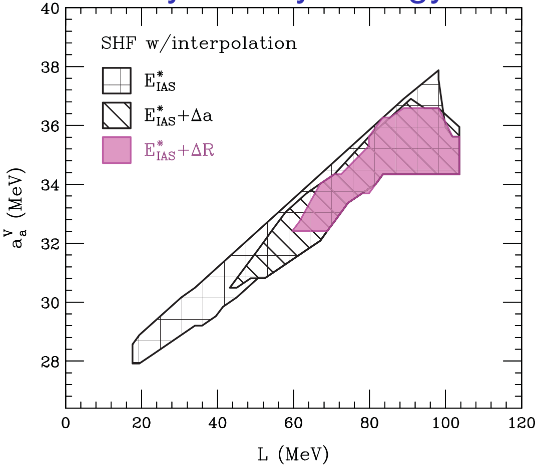


148 Skyrme parametrizations

Skyrme Interpolations in (a_a^V, L) Plane



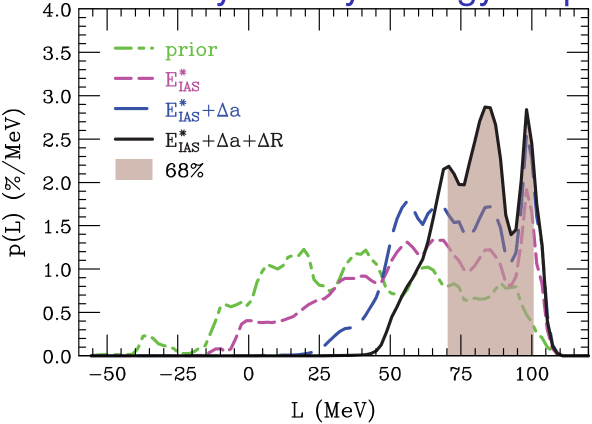
Constraints on Symmetry-Energy Parameters



68% contours for probability density

E^*_{IAS} - from excitations to isobaric analog states in PD&Lee NPA922(14)1

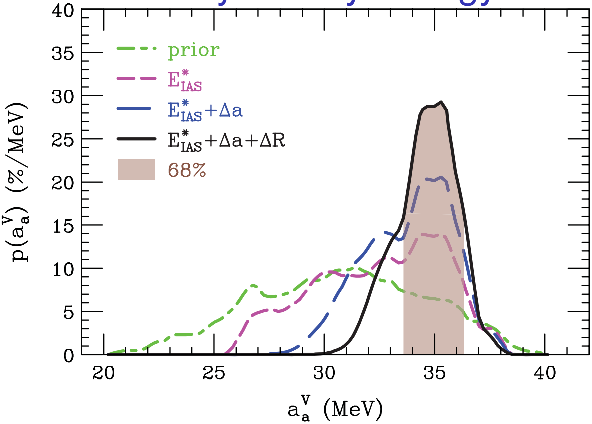
Likelihood f/Symmetry-Energy Slope



E^*_{IAS} - from excitations to isobaric analog states in PD&Lee NPA922(14)1

Oscillations in prior of no significance
- represent availability of Skyrme parametrizations

Likelihood f/Symmetry-Energy Value



E^*_{IAS} - from excitations to isobaric analog states in PD&Lee NPA922(14)1

Oscillations in prior of no significance
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Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density
- For large A, displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy
- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions
- Such an analysis produces thick isovector aura
  $\Delta R \sim 0.9\,\mathrm{fm}$!
- Symmetry energy is stiff!
  $L = (70-100)\,\mathrm{MeV}$, $a_a^V = (33.5-36.5)\,\mathrm{MeV}$ at 68% level

PD&Lee NPA818(09)36 NPA922(14)1; PD, Singh *et al*

US PHY-1403906 + Indo-US Grant

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